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Lateral Transport in a Fluidized-Packed Bed: Part I. Solids Mixing

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A theory was developed to relate the average velocity of a fluidized particle to the fluidizing gas velocity and the minimum fluidization velocity. The development of this relationship was based on a model for particle movement caused by gas bubbles rising through the fluidized bed. The diffusivity for lateral solids mixing in a fluidized-packed bed can then be determined from the average particle velocity and the diameter of the fixed packing by the random walk theory.

The advantages of a fluidized bed over a packed bed as a reactor lie in the movement of the fluidized particles. The moving particles of the fluidized bed transport essentially all of the heat (*1, 12, 13, 14*), whereas heat transfer in the packed bed is essentially by the gas eddies (*7*). Because the volumetric heat capacity of solid particles is much greater than that of gas, the heat content and hence heat transport by fluidized particles is greater. The fluidized particles also disturb surface gas films to increase mass transfer (*14*). The fluidized-packed bed provides a means of utilizing the advantages of fluidized particles in packed-bed reactors. Chemical reactors employing inert solids fluidized in the interstices of a static packed bed of larger bodies undergoing an exothermic gaseous reaction have been demonstrated to provide good reaction control (*5, 6*).

Particle movement in a fluidized-packed bed is in the form of streams of aggregates flowing vertically which are laterally deflected by the fixed packing. The mean distance $\overline{\Delta X}$ that a particle is deflected by a sphere in the x direction (the direction of diffusion) is D_p/π, and the time θ between contacts of the fluidized particle at velocity u with the fixed packing arranged rhombohedrally is $0.909\ D_p/u$ (*4*). The lateral solids mixing diffusion coefficient D can be related to $\overline{\Delta X}$ and θ by Einstein's (*3*) random walk diffusion equation to yield

$$D = 1/2\ (\overline{\Delta X^2}/\theta) = 1/2 \frac{u}{0.909\ D_p} \left(\frac{D_p}{\pi}\right) = 0.0558\ D_p u \quad (1)$$

The random walk theory indicates the mechanism of lateral particle movement and relates the rate of lateral solids mixing to the size of the fixed packing but does not indicate the effect of the physical properties of the fluidized particles and fluidizing gas on the solids mixing. A model is presented which reveals the effect of gas and solids properties on the mixing by relating the average particle velocity to the fluidizing gas velocity and minimum fluidization velocity. Experimental data are then correlated in terms of the model.

MODEL FOR RELATING AVERAGE PARTICLE VELOCITY TO THE FLUIDIZING GAS VELOCITY

The analysis of Rowe (*8*) and co-workers has shown that the particles are transported by the drag forces in the wake of bubbles rising through the fluidized bed. This concept was used as the basis of a model for particle movement. The model adopted here is defined by these assumptions:

1. Gas flow in excess of the minimum velocity required for fluidization forms bubbles; that is the volumetric rate of bubble formation is $(W - W_{mf})$.

2. The ratio of the volume of particles moved by the bubbles to the volume of the bubbles is a constant F' for a given particle-gas system; that is the volumetric rate of particles in motion is equal to $F'\ (W - W_{mf})$ for particles of a particular size. This assumption is equivalent to asserting that F' is independent of bubble size as well as frequency.

From the above assumptions the volumetric rate of particles per unit area dragged by gas flow through the bubble (average particle velocity) can be expressed as

$$u = F' \left[\frac{W - W_{mf}}{S\ \epsilon_p} \right] = F'\ (V - V_{mf}) \quad (2)$$

The volume F' of particles moved will depend upon the force exerted on the particles (drag force) and the weight of each particle. Rowe and Henwood (*10*) have shown that the drag coefficient for a particle in an array of surrounding particles is 68.5 times the drag coefficient for an isolated particle. On this basis, Rowe (*9*) has shown that

TABLE 1. DRAG RATIOS FOR DIFFERENT COPPER-NICKEL PARTICLE SIZES

Fluidizing gas: N_2 at 20°C.
$\rho = 0.0724$ lb./cu. ft.
$\mu = 0.0175$ centipoise

Particle size Mesh	Particle size Inches	Particle weight M_p, lb.	Minimum fluidization velocity V_{mf}, ft./sec.	V_{mf} (calc.), ft./sec.	Vel. V, ft./sec.	$\frac{dV\rho}{\mu}$	$68.5 \frac{C_D \rho V^2}{2g} \frac{d^2}{4} \frac{1}{M_p}$	Relative drag ratio	Relative minimum fluidization velocity $\frac{V_{mf\,0.0054}}{V_{mf}}$
−140 + 170	0.0038	0.0922×10^{-7}	0.122	0.122	0.28	0.54	2.46	1.97	2.24
					1.00	1.96	10.13	1.95	
					2.00	3.92	27.5	1.89	
					3.00	5.88	37.1	1.87	
					4.00	7.84	52.5	1.83	
−100 + 120	0.0054	0.265×10^{-2}	0.276	0.230	0.28	0.77	1.25	1.00	1.00
					1.00	2.78	5.21	1.00	
					2.00	5.65	11.9	1.00	
					3.00	8.35	19.9	1.00	
					4.00	11.1	28.7	1.00	
−60 + 70	0.00905	1.75×10^{-7}	0.706	0.578	0.28	1.29	0.462	0.364	0.391
					1.00	4.67	2.04	0.391	
					2.00	9.34	4.83	0.406	
					3.00	14.0	8.20	0.412	
					4.00	18.7	12.05	0.420	
−40 + 50	0.0141	4.72×10^{-7}	1.56	1.072	0.28	2.01	0.201	0.161	0.177
					1.00	7.27	0.934	0.174	
					2.00	14.5	2.29	0.193	
					3.00	21.8	3.97	0.200	
					4.00	29.1	5.96	0.208	

the minimum fluidization velocity can be calculated in the same manner as the terminal falling velocity by correcting the drag coefficient for an isolated sphere by this factor of 68.5. In Table 1 the value for the drag force on the particle divided by particle weight is given for each particle size as calculated for increasing gas velocity. In Table 1 also a comparison is made of the drag on each particle relative to the 0.0054-in. particles for each value of gas velocity. That is at each gas velocity the relative drag ratio $(\text{drag})_d/(\text{drag})_{0.0054}$ was calculated. Ratios of the minimum fluidization gas velocities for the 0.0054-in. particles to the minimum fluidization velocities for the other particles were also made. The ratios of the minimum fluidization velocities are in close agreement with the relative drag ratios for a gas velocity range of 0.28 to 4.0 ft./sec. Also given in Table 1 are minimum fluidization velocities calculated by the method presented by Rowe. Effects caused by the presence of the fixed packing were neglected in the calculation of minimum fluidization velocities. The experimental minimum fluidization velocities were based on the superficial velocity divided by the fixed packing void fraction ($\epsilon_p = 0.415$ for ⅜-in. spherical packing). The experimentally determined minimum fluidization velocities will be the basis for estimating the relative volume of material dragged in the bubble wake.

The volumetric rate of particles/unit area moved by the dragging action of bubbles is given by the product of the excess gas velocity $(V - V_{mf})$ for the given size particles times the constant F determined for the reference particle system (−100 +200 mesh, 0.0054 in. mean diameter) times the factor $V_{mf\,0.0054}/V_{mf}$ correcting for relative drag ratios. Therefore, with the −100 +120 mesh particles used as a basis, the average particle velocity is

$$u = F\left[\frac{V_{mf\,0.0054}}{V_{mf}}\right](V - V_{mf}) \tag{3}$$

Since the minimum fluidization velocity for the −100 +200 mesh particles is a constant, Equation (3) can be simplified to

$$u = K\left[\frac{V - V_{mf}}{V_{mf}}\right] \tag{4}$$

The rate of fluidized solids movement depends not only on the proportion of particles moved but also on the absolute velocity of the bubble rising through the bed. The above equation implies that the velocity of the bubbles rising through beds of different size fluidized particles would be the same. Relative bubble velocities were estimated by determining bubble residence time from bed expansion and volumetric gas flow. The bubble velocities at the same $V - V_{mf}$ for the 0.0054, 0.00905, and 0.0141 in. fluidized particles were the same. The bubble velocity for the bed of 0.0038-in. particles was about 10% less relative to the bubble velocity for the bed of 0.0054-in. particles. However, as concluded by Davies and Robinson (2), at present no generally applicable correlation of expansion for aggregatively fluidized beds exists. Since accounting for the relative bubble velocities makes little change in the value for u, and since a general correlation of bed expansion does not exist for predicting this effect, this refinement will be neglected in the model.

A correlation of D is suggested by substitution of u in Equation (4) into Equation 1:

$$D = 0.0558\, D_p\, K\left[\frac{V - V_{mf}}{V_{mf}}\right] \tag{5}$$

CORRELATION OF DATA WITH THEORY

On Figure 1 are solids mixing diffusivity data for four different sizes of copper-nickel shot fluidizing in the interstices of ⅜ in. spherical packing. Measurements were

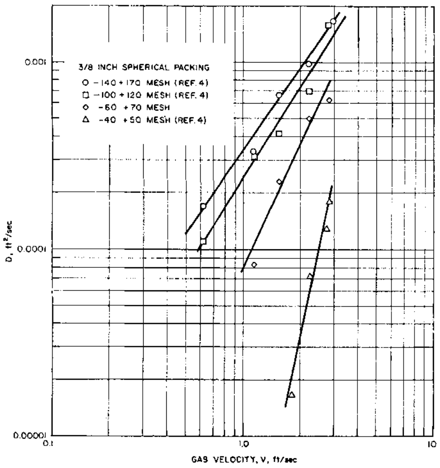

Fig. 1. Solids mixing diffusivities for different sizes of copper-nickel shot fluidizing in voids of ⅜-in. spherical packing.

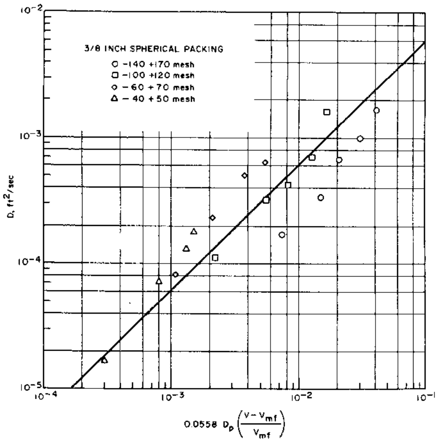

Fig. 2. General correlation of lateral solids mixing in a fluidized-packed bed.

made in a rectangular column 7 in. long, 1 13/16 in. wide, and 10 in. high. The solids mixing diffusivities were determined by placing pure nickel shot in the column on one side of a centrally located partition and pure copper shot on the other side. The partition was removed, and the copper-nickel particles were then fluidized for a known time interval. Bed samples were taken at various distances along the length of the column after fluidization and were analyzed by magnetic separation for the concentrations of nickel and copper. From the time of mixing and the concentration profile mixing diffusivities could be determined (*11*). The column and analytical techniques have been described in detail previously (*4*).

A plot of the experimentally obtained diffusivities vs. 0.0558 D_p $(V - V_{mf}/V_{mf})$ is made on Figure 2. The points deviate up to a factor of 2 from the solid line, but when one considers that a range of two orders of magnitude is covered, the theory does quite well. Also to be noted is that the solid line has a slope of unity and that for each particle size the points appear to have about the same slope as the solid line. This simple theory neglects any effects that may be associated with interparticle attractive forces and friction which could account for the deviation of each particle size. The correlation yields a value for K of 0.061. From Figure 2 the value F is then found to be 0.22. Therefore, for the 0.0054-in. diameter particles the amount of particles dragged is approximately one-fifth the bubble volume.

In a previous paper (*4*) an empirical correlation was given. The scatter was somewhat reduced, but the lack of first principles in developing this correlation prevented it from being extended to different fluidizing materials (rather than just copper and nickel spherical shot) and different fluidizing gases. The use of a model for the correlation on Figure 2 gives a basis for extension to other fluidized systems. The method of Rowe (*9*) can be used for predicting minimum fluidization velocities if minimum fluidization velocity data for a particular fluidization system are not available. Rowe's method was applied to the materials described here, and minimum fluidization velocities were predicted fairly well (Table 1). This model is further verified by measurements of thermal diffusivities described in Part II.

NOTATION

C_D = drag coefficient
d = fluidized particle diameter, ft.
D = diffusivity, sq. ft./sec.
D_p = fixed packing diameter, ft.
F = volume at particles moved/bubble volume for 0.0054 in. diameter reference particle system
F' = volume of particles moved/bubble volume
M_p = particle weight, lb.
g = gravitational constant
K = $(F)\,(V_{mf\ 0.0054})$, ft./sec.
S = column cross sectional area, sq. ft.
u = average particle velocity, ft./sec.
V = fluidizing gas velocity corrected for packing voidage, ft./sec.
V_{mf} = minimum fluidization velocity, ft./sec.
$V_{mf\ 0.0054}$ = minimum fluidization velocity for −100 +200 mesh copper-nickel shot, 0.276 ft./sec.
W = volumetric gas rate, cu. ft./sec.
W_{mf} = volumetric gas rate required for minimum fluidization, cu. ft./sec.

Greek Letters

$\overline{\Delta X}$ = mean free path of fluidized particle, ft.
ϵ_p = voidage of fixed packing
θ = time, sec.
μ = viscosity of gas, ft.-lb./sec.
ρ = density of gas, lb./cu. ft.

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